Functional Equation of the Rate of Inflation

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Abstract: This short note aims to introduce a rule which admits to compute interest in any time per any time, rate of inflation per any time in any moment, if the rate of interest or the rate of inflation by unity of time is an arbitrary integrable function.
The main result is the generalization of the well known rule $\iota(t)+1=\frac{\mathrm{X}(0)}{\mathrm{X}(t)}=\prod_{i=0}^{n-1}(I_i+1)^{(t_{i+1}-t_i)}$, which holds for the piecewise constant approximation of the rate of inflation with the values I_i to rule $\iota(t)+1=\frac{\mathrm{X}(0)}{\mathrm{X}(t)}=e^{\left(\int_0^t \ln(1+\iota(u))\,\mathrm{d}\,u\right)}$, which should be used for arbitrary integrable function ι.
The usage of the rule is demonstrated on the examples based on real data of index of prices of non regulated prices in Czech republic.
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1. Introduction: Let $\mathrm{CPI}_i(t)$ be the index of prices in time t, let $X(t)$ be the value of an objective function (the real value of a unit of money), let $\iota(t)$ be rate of inflation per time $\langle 0,1\rangle$. Then

$$\frac{X(t_0)}{X(t_1)}=\frac{\mathrm{CPI}(t_1)}{\mathrm{CPI}(t_0)}=1+\iota(\langle t_0,t_1\rangle) \tag{1}$$

We can measure CPI in any time, consequently we can fix $\iota(\langle t_0,t_1\rangle)$ in an arbitrary interval $\langle t_0,t_1\rangle$

1.1. Definition: If $\iota(\langle t_0,t_1\rangle)$ in some interval $\langle T_0,T_1\rangle\supset\langle t_0,t_1\rangle$ depends only on the length t_1-t_0 of the interval $\langle t_0,t_1\rangle$ but not on the origin t_0 of the interval we call the inflation constant (on interval $\langle T_0,T_1\rangle$).

1.2. Lemma: The inflation is constant, if and only if the CPI is exponential function $x\mapsto a\cdot e^{bt}$.

Let the inflation is constant and let us denote $\iota(t_1-t_0)=\iota(\langle t_0,t_1\rangle)$, then it holds

$$1+\iota(t_i)=(1+\iota(t_j))^{\frac{t_j}{t_i}} \tag{2}$$

especially

$$1+\iota(t)=(1+\iota(1))^t \tag{3}$$

and the real value (value of the objective function) in the time t is

$$X(t)=\frac{X(0)}{(1+\iota)^t} \tag{4}$$

Let the inflation depends differentiably on the time in a time t_0 almost everywhere. We define the rate of inflation per unity of time in time t_0 as rate of constant inflation which has first order contact with the inflation in question almost everywhere and on the rest of the points we can define it by one sided limits.

More precisely: The solution of equations:

$$\begin{aligned} F(x)=Ae^{Bx}&=\mathrm{CPI}(x)\\ \mathrm{D}(F)(x)=\mathrm{D}\left(Ae^{Bx}\right)=ABe^{Bx}&=\mathrm{D}(\mathrm{CPI})(x)\end{aligned} \tag{5}$$

is

$$
\begin{aligned}
A &= \mathrm{CPI}(x)\left(e^{\frac{(\mathrm{D}(\mathrm{CPI})(x))x}{\mathrm{CPI}(x)}}\right)^{-1} \\
B &= \frac{\frac{\partial}{\partial x}(\mathrm{CPI})(x)}{\mathrm{CPI}(x)}
\end{aligned}
\tag{6}
$$

Consequently

$$
\iota = \iota(1) = x \mapsto \frac{F(x+1)}{F(x)} - 1 = x \mapsto e^{\frac{\mathrm{D}(\mathrm{CPI})(x)}{\mathrm{CPI}(x)}} - 1 = x \mapsto e^{\mathrm{D}(\ln\circ\mathrm{CPI})(x)} - 1 \tag{7}
$$

1.3. Definition: Let CPI be the index of prices, then the mapping

$$
x \mapsto e^{\frac{\mathrm{D}(\mathrm{CPI})(x)}{\mathrm{CPI}(x)}} - 1 \tag{8}
$$

is the rate of inflation per unity of time (in time x).

2. Functional equation of rate of inflation:

The most general case considered so far is that one with CPI_i piecewise exponential function $x \mapsto a_i e^{b_i x}$ with the constant a_i, b_i on an interval $\langle t_i, t_{i+1}\rangle$, $(t_0 = 0)$, then the rate of inflation per unity of time is piecewise constant function and the rate of inflation per time $\langle 0, t\rangle$ is $\prod_{\{i|t_i\le t\}} \frac{\mathrm{CPI}_{i+1}}{\mathrm{CPI}_i} - 1$

Let us suppose that CPI is the integrable function, ι is the rate of inflation per unity of time (integrable function), X the objective function (the opposite value of CPI in the suitable rescaling)

If an inflation is constant and if its rate of time of duration t is equal to $\iota(t)$, then

$$
1 + \iota(t) = (1 + \iota(1))^t = \frac{\mathrm{X}(0)}{\mathrm{X}(t)}. \tag{9}
$$

If an inflation is piecewise constant (the rate of the inflation per time unit is piecewise constant function) and if the rate of inflation per a unit of time in every point of interval $\langle t_i,\, t_{i+1}\rangle$; $i = 0 \dots n$ is equal I_i,then for the rate of inflation of time t $\langle t_0,\, t_n\rangle$ holds

$$
1 + \iota(\langle t_0,\, t_n\rangle) = (\prod_{i=0}^{n-1} (I_i + 1)^{(t_{i+1} - t_i)}) = \frac{\mathrm{X}(t_0)}{\mathrm{X}(t_n)}. \tag{10}
$$

This note aims to find the rate of inflation in the time t per a time period $\langle t_0,\, t_1\rangle$ ($\langle 0,\, t\rangle$, after suitable rescaling), in the case that the rate of inflation per unit of time is an arbitrary integrable function.

2.4. Theorem: If the rate of inflation per unit of time t equals $\iota(t)$, then the rate of inflation per time $\langle 0,\, t\rangle$ (plus one) is equal to

$$
\iota(\langle t_0,\, t_n\rangle) = \frac{\mathrm{X}(t_0)}{\mathrm{X}(t_n)} - 1 = e^{\left(\int_{t_0}^{t_n} \ln(1+\iota(u))\,\mathrm{d}\,u\right)} - 1 \tag{11}
$$

and the equation

$$
\frac{\mathrm{X}(t_0)}{\mathrm{X}(t_n)} = \prod_{i=0}^{n-1} (I_i + 1)^{(t_{i+1} - t_i)} \tag{12}
$$

is a special case of the previous one for the piecewise constant function ι with values I_i on intervals $\langle t_i, t_{i+1}\rangle$.

2.5. Example

Let us suppose, that the rate of unit of time inflation was 0.1 in time 0 and rate of unit of time inflation was 0.2 in time 1. Then the rate of inflation should be changed in time $\langle 0, 1\rangle$ between booth values for instance in this four following ways:

$$
\iota_1 := \begin{cases} 0.1, & \text{if } x < 1 \\ 0.2, & \text{if } x \ge 1 \end{cases}, \quad \iota_2 := \frac{u^2}{10} + 0.1, \quad \iota_3 := \frac{u}{10} + 0.1, \quad \iota_4 := \begin{cases} 0.1, & \text{if } x \le 0 \\ 0.2, & \text{if } x > 0 \end{cases} \tag{13}
$$

The questions is: What is the real value $X(1)$ in time 1 if the real value $X(0)$ of it in time 0 was 100?

In general, we have

$$X(1) = \frac{X(0)}{e^{\left(\int_0^1 (\ln(1+\iota(u)) \, \mathrm{d}\, u\right)}} \tag{14}$$

So, in our four cases we obtain:

$$\iota = \iota_1 = 0.1 \colon X(1) = \frac{100}{e^{\left(\int_0^1 \ln(1.1) \, \mathrm{d}\, u\right)}} = 90.9090909\ldots = \frac{100}{1+0.1}$$

$$\iota = \iota_2 = u \mapsto \frac{1}{10}u^2 + 0.1 \colon X(1) = \frac{100}{e^{\left(\int_0^1 \ln(1.1+u^2/10) \, \mathrm{d}\, u\right)}} = 88.26551047\ldots$$

$$\iota = \iota_3 = u \mapsto \frac{1}{10}u + 0.1 \colon X(1) = \frac{100}{e^{\left(\int_0^1 \ln(1.1+u/10) \, \mathrm{d}\, u\right)}} = 86.98393756\ldots$$

$$\iota = \iota_4 = 0.2 \colon X(1) = \frac{100}{e^{\left(\int_0^1 \ln(1.2) \, \mathrm{d}\, u\right)}} = 83.33333333\ldots = \frac{100}{1+0.2}$$

2.6. Proof:

1) The equation $\frac{\mathrm{x}(0)}{\mathrm{x}(t)} = e^{\left(\int_0^t \ln(1+\iota(u)) \, \mathrm{d}\, u\right)}$ gives the supposed results with the constant inflation: let us suppose, that $\iota(t) = I$ is constant:

$$\frac{\mathrm{X}(0)}{\mathrm{X}(t)} = e^{\left(\int_0^t \ln(1+I) \, \mathrm{d}\, u\right)} = e^{(t \cdot \ln(1+I))} = e^{\left(\ln(1+I)^t\right)} = (1+I)^t \tag{15}$$

q. e. d.

2) Now let us suppose, that ι is piecewise constant and that it has value I_i in every point of the interval $I_i = (t_i,\, t_{i+1})\,;\, i = 0 \ldots n$. Let χ_A be the characteristic function of the set A, then $\iota(t) = \sum \chi_{(t_i, t_{i+1})} \cdot I_i$

$$\frac{\mathrm{X}(0)}{\mathrm{X}(t)} = e^{\left(\int_0^t \ln(1+\chi_{(t_i,t_{i+1})} \cdot I_i \, \mathrm{d}\, u\right)} = e^{\left(\sum_{i=0}^{n-1} (t_{i+1} \ln(1+I_i) - t_i \ln(1+I_i))\right)} \tag{16}$$

We shall use

$$e^{\left(\sum_{i=0}^{n-1} \zeta_i\right)} = \prod_{i=0}^{n-1} e^{\zeta_i}$$

hence

$$e^{\left(\sum_{i=0}^{n-1} (t_{i+1} \ln(1+I_i) - t_i \ln(1+I_i))\right)} = \prod_{i=0}^{n-1} (1+I_i)^{(t_{i+1}-t_i)}$$

consequently

$$\frac{\mathrm{X}(0)}{\mathrm{X}(t)} = \prod_{i=0}^{n-1} e^{(t_{i+1}-t_i)\ln(1+I_i)} = \prod_{i=0}^{n-1} e^{\ln(1+I_i)^{(t_{i+1}-t_i)}} = \prod_{i=0}^{n-1} (1+I_i)^{(t_{i+1}-t_i)} \tag{17}$$

q. e. d.

3. Example with the real data of index of prices of non-regulated prices in Czech republic:.

We choose the year as a unity of time. We measured the index of non-regulated prices in every moment $P = \left\{1993 + \frac{i}{12}\right\}$, where i is a natural number less or equal to 120. The values are relative. As a result of the measurement, CPI is a function cutting the points: $(1993 + \frac{1}{12};\ 91.76)$, $(1993 + \frac{1}{6};\ 92.97)$, $(1993 + \frac{1}{4};\ 93.54)$, $(1993+\frac{1}{3};\ 93.94)$, $(1993+\frac{5}{12};\ 94.32)$, $(1993+\frac{1}{2};\ 94.62)$, $(1993+\frac{7}{12};\ 95.42)$, $(1993+\frac{2}{3};\ 96.10)$, $(1993+\frac{3}{4};\ 97.53)$, $(1993+\frac{5}{6};\ 98.62)$, $(1993+\frac{11}{12};\ 99.17)$, $(1994;\ 100.0)$, $(1994+\frac{1}{12};\ 100.76)$, $(1994+\frac{1}{6};\ 101.12)$, $(1994+\frac{1}{4};\ 101.40)$, $(1994 + \frac{1}{3};\ 101.88)$, $(1994 + \frac{5}{12};\ 102.27)$, $(1994 + \frac{1}{2};\ 103.45)$, $(1994 + \frac{7}{12};\ 103.81)$, $(1994 + \frac{2}{3};\ 104.62)$, $(1994+\frac{3}{4};\ 106.13)$, $(1994+\frac{5}{6};\ 107.45)$, $(1994+\frac{11}{12};\ 108.46)$, $(1995;\ 109.23)$, $(1995+\frac{1}{12};\ 110.57)$, $(1995+$

$\frac{1}{6}$; 111.62), (1995 + $\frac{1}{4}$; 111.97), (1995 + $\frac{1}{3}$; 112.72), (1995 + $\frac{5}{12}$; 113.26), (1995 + $\frac{1}{2}$; 114.13), (1995 + $\frac{7}{12}$; 113.47), (1995+$\frac{2}{3}$; 113.41), (1995+$\frac{3}{4}$; 114.41), (1995+$\frac{5}{6}$; 115.20), (1995+$\frac{11}{12}$; 116.09), (1996; 116.82), (1996+$\frac{1}{12}$; 118.97), (1996 + $\frac{1}{6}$; 119.62), (1996 + $\frac{1}{4}$; 120.43), (1996 + $\frac{1}{3}$; 121.15), (1996 + $\frac{5}{12}$; 121.96), (1996 + $\frac{1}{2}$; 123.06), (1996 + $\frac{7}{12}$; 123.18), (1996 + $\frac{2}{3}$; 122.67), (1996 + $\frac{3}{4}$; 123.04), (1996 + $\frac{5}{6}$; 123.76), (1996 + $\frac{11}{12}$; 124.44), (1997; 125.23), (1997+$\frac{1}{12}$; 126.28), (1997+$\frac{1}{6}$; 126.71), (1997+$\frac{1}{4}$; 126.85), (1997+$\frac{1}{3}$; 127.45), (1997+$\frac{5}{12}$; 127.60), (1997 + $\frac{1}{2}$; 129.42), (1997 + $\frac{7}{12}$; 129.67), (1997 + $\frac{2}{3}$; 130.77), (1997 + $\frac{3}{4}$; 131.58), (1997 + $\frac{5}{6}$; 132.33), (1997+$\frac{11}{12}$; 133.01), (1998; 133.75), (1998+$\frac{1}{12}$; 135.76), (1998+$\frac{1}{6}$; 136.71), (1998+$\frac{1}{4}$; 136.85), (1998+$\frac{1}{3}$; 137.12), (1998 + $\frac{5}{12}$; 137.26), (1998 + $\frac{1}{2}$; 137.81), (1998 + $\frac{7}{12}$; 137.53), (1998 + $\frac{2}{3}$; 137.12), (1998 + $\frac{3}{4}$; 137.25), (1998+$\frac{5}{6}$; 136.84), (1998+$\frac{11}{12}$; 136.43), (1999; 136.02), (1999+$\frac{1}{12}$; 136.70), (1999+$\frac{1}{6}$; 136.57), (1999+$\frac{1}{4}$; 136.29), (1999 + $\frac{1}{3}$; 136.84), (1999 + $\frac{5}{12}$; 136.70), (1999 + $\frac{1}{2}$; 136.98), (1999 + $\frac{7}{12}$; 136.98), (1999 + $\frac{2}{3}$; 137.11), (1999+$\frac{3}{4}$; 136.98), (1999+$\frac{5}{6}$; 136.98), (1999+$\frac{11}{12}$; 137.39), (2000; 138.21), (2000+$\frac{1}{12}$; 139.04), (2000+$\frac{1}{6}$; 139.32), (2000 + $\frac{1}{4}$; 139.32), (2000 + $\frac{1}{3}$; 139.32), (2000 + $\frac{5}{12}$; 139.74), (2000 + $\frac{1}{2}$; 140.71), (2000 + $\frac{7}{12}$; 141.42), (2000+$\frac{2}{3}$; 141.70), (2000+$\frac{3}{4}$; 141.56), (2000+$\frac{5}{6}$; 141.98), (2000+$\frac{11}{12}$; 142.13), (2001; 142.41), (2001+$\frac{1}{12}$; 143.26), (2001 + $\frac{1}{6}$; 143.26), (2001 + $\frac{1}{4}$; 143.26), (2001 + $\frac{1}{3}$; 143.84), (2001 + $\frac{5}{12}$; 144.99), (2001 + $\frac{1}{2}$; 146.87), (2001 + $\frac{7}{12}$; 147.90), (2001 + $\frac{2}{3}$; 147.46), (2001 + $\frac{3}{4}$; 145.98), (2001 + $\frac{5}{6}$; 145.84), (2001 + $\frac{11}{12}$; 145.69), (2002; 145.98), (2002+$\frac{1}{12}$; 147.30), (2002+$\frac{1}{6}$; 147.30), (2002+$\frac{1}{4}$; 147.0), (2002+$\frac{1}{3}$; 147.30), (2002+$\frac{5}{12}$; 147.15), (2002 + $\frac{1}{2}$; 146.71), (2002 + $\frac{7}{12}$; 147.15), (2002 + $\frac{2}{3}$; 146.85), (2002 + $\frac{3}{4}$; 145.82), (2002 + $\frac{5}{6}$; 145.82), (2002 + $\frac{11}{12}$; 145.68), (2003; 147.97). Now we are going to approximate the values of CPI with the function. There are a lot of ways of how to do it. We shall measure the accuracy of approximation as the sum of squares of differences between values of function and measured values in points where the measurement is made. Trivial approximation is to put together points by abscissas. We obtain piecewise affine function: and the accuracy is equal to 0. In this case the rate of the year inflation ι is the function:

$$\iota(t) = \begin{cases} e^{14.52058082\,(14.52058082\,t-28847.75831)^{-1}} - 1 & t \in \langle 1993.00000, 1993.08333\rangle \\ e^{6.840273611\,(6.840273611\,t-13540.26521)^{-1}} - 1 & t \in \langle 1993.08333, 1993.16666\rangle \\ e^{4.799616031\,(4.799616031\,t-9472.894168)^{-1}} - 1 & t \in \langle 1993.16666, 1993.25000\rangle \\ e^{4.560182407\,(4.560182407\,t-8995.643826)^{-1}} - 1 & t \in \langle 1993.25000, 1993.33333\rangle \\ e^{3.600144006\,(3.600144006\,t-7081.966879)^{-1}} - 1 & t \in \langle 1993.33333, 1993.41666\rangle \\ e^{9.599232061\,(9.599232061\,t-19040.64915)^{-1}} - 1 & t \in \langle 1993.41666, 1993.50000\rangle \\ e^{8.160326413\,(8.160326413\,t-16172.19129)^{-1}} - 1 & t \in \langle 1993.50000, 1993.58333\rangle \\ e^{17.16068643\,(17.16068643\,t-34115.15901)^{-1}} - 1 & t \in \langle 1993.58333, 1993.66666\rangle \\ e^{13.07895368\,(13.07895368\,t-25977.54380)^{-1}} - 1 & t \in \langle 1993.66666, 1993.75000\rangle \\ e^{6.600264011\,(6.600264011\,t-13060.65643)^{-1}} - 1 & t \in \langle 1993.75000, 1993.83333\rangle \\ e^{9.960398416\,(9.960398416\,t-19760.20401)^{-1}} - 1 & t \in \langle 1993.83333, 1993.91666\rangle \\ e^{9.120364815\,(9.120364815\,t-18085.24781)^{-1}} - 1 & t \in \langle 1993.91666, 1993.99999\rangle \\ e^{4.320172807\,(4.320172807\,t-8513.664947)^{-1}} - 1 & t \in \langle 1993.99999, 1994.08332\rangle \\ e^{3.359731222\,(3.359731222\,t-6598.464123)^{-1}} - 1 & t \in \langle 1994.08332, 1994.16666\rangle \\ e^{5.760230409\,(5.760230409\,t-11385.45902)^{-1}} - 1 & t \in \langle 1994.16666, 1994.24999\rangle \\ e^{4.680187207\,(4.680187207\,t-9231.582863)^{-1}} - 1 & t \in \langle 1994.24999, 1994.33332\rangle \\ \vdots & \end{cases}$$

and the rate of inflation per the time interval $\langle 1993.5, 1994\rangle$ equals

$$\frac{\mathrm{CPI}(1994)}{\mathrm{CPI}(1993.5)} - 1 = e^{\int_{1993.5}^{1994} \ln(1+\iota(z))\,\mathrm{d}\,z} = 0.05596 \tag{18}$$

We can use more sophisticated approximation, for instance: let us approximate a trend by function $151.558469453 + 53.8746490595\cdot(5/12)^{(x-1992)} - 108.487078769\cdot(3/4)^{(x-1992)}$ which is the best approximation of the measured values by means of pair of functions out of the set $\left\{(i/12)^{(x-1992)};\ (i/12)^{(x-1993)};\ (x-1992)^{(i/12)};\ (x-1993)^{(i/12)};\ \ln(x-1992);\ \ln(x-1991)\right\}_{i=1}^{24}$ and constant (accuracy (the sum of squares of distances between values of the functions and the measured values equals 237.324366060)) Then let us approximate the rest by linear combination of functions

$$\{\sin(\pi x/i); \cos(\pi x/i); x\sin(\pi x/i); x\cos(\pi x/i)\}_{i=1}^{5} \tag{19}$$

Let us omit 15 of the functions from this set of functions, the absence of which brings the least loss of accuracy. We obtain the following approximation of CPI: $\mathrm{CPI}(t) = 151.558469453 + 53.8746490595 \cdot (5/12)^{(x-1992)} - 108.487078769 \cdot (3/4)^{(x-1992)} - 0.08688205539 + 438.678532386 \cdot \sin(1/5 \cdot x \cdot \pi) - 0.219746118617 \cdot x \cdot \sin(1/5 \cdot x \cdot \pi) - 0.392448110715 \cdot x \cdot \cos(1/2 \cdot x \cdot \pi) + 0.000156347532197 \cdot x \cdot \sin(x \cdot \pi) + 782.647022069 \cdot \cos(1/2 \cdot x \cdot \pi)$ (accuracy (the sum of squares of distances between values of the functions and the measured values equals 84.8709833772)) Then the rate of inflation per year has in the time x the value

$$e^{53.8746490595\,\frac{A}{B}}$$

$$\begin{aligned}
A = \left(\frac{5}{12}\right)^{x-1992} \ln(\frac{5}{12}) &- 108.487078769\,(3/4)^{x-1992}\ln(3/4) + 87.73570648\,\cos(1/5\,x\pi)\pi \\
&-0.219746118617\,\sin(1/5\,x\pi) - 0.04394922372\,x\cos(1/5\,x\pi)\pi \\
&-0.392448110715\,\cos(1/2\,x\pi) + 0.1962240554\,x\sin(1/2\,x\pi)\pi \\
&+0.000156347532197\,\sin(x\pi) + 0.000156347532197\,x\cos(x\pi)\pi \\
&-391.3235111\,\sin(1/2\,x\pi)\pi \\
B = 151.4715874 + 53.8746490595\left(\frac{5}{12}\right)^{x-1992} &- 108.487078769\,(3/4)^{x-1992} + 438.678532386\,\sin(1/5\,x\pi) \\
&-0.219746118617\,x\sin(1/5\,x\pi) - 0.392448110715\,x\cos(1/2\,x\pi) \\
&+0.000156347532197\,x\sin(x\pi) + 782.647022069\,\cos(1/2\,x\pi)^{-1}
\end{aligned}$$

and

$$\frac{\mathrm{CPI}(1994)}{\mathrm{CPI}(1993.5)} - 1 = e^{\int_{1993.5}^{1994} \ln(1+\iota(z))\,\mathrm{d}\,z} = 0.04737 \tag{20}$$

But we can approximate the rate of inflation directly. If $(t_{n_i})_{i\in J}$, $i < j \Rightarrow t_i < t_j$ are the moments, where CPI is measured, then we have to fulfil the equation:

$$\forall i \in J\colon \int_{t_i}^{t_{i+1}} \ln\left(1 + \iota(t)\right) \mathrm{d}\,t = \ln\circ\mathrm{CPI}(t_{n+1}) - \ln\circ\mathrm{CPI}(t_n) \tag{21}$$

and the accuracy of approximation should be measured as the measure of unreliableness of the equation.

More precisely: Let us choose the function:

$$\begin{aligned}
\iota\colon x \longmapsto 0.04944210 + 0.009094682\,\cos(x\pi) + 0.01215932\,\sin(2\,x\pi) + \\
+0.01823419\,\sin(4\,x\pi) + 0.05263253\,\sin(1/3\,x\pi) - \\
-0.009667420\,\cos(1/3\,x\pi) - 0.07212055\,\cos(1/4\,x\pi) + 0.02834145\,\sin(1/2\,x\pi)\,.
\end{aligned} \tag{22}$$

The question is, how accurately this function corresponds to the measured values. The interesting thing is, that the function do not approximate measured values, but some other values, which depend on the measured ones. And the attitude towards this two values is given by (see. (21)). So if we can compare the exactness of approximation with the same measure as we did in previous cases, we have to compute the number

$$\sum_{i=1}^{120}\left(e^{\int_{t_i}^{t_{i+1}} \ln(1+\iota(t))\,\mathrm{d}\,t} - \frac{\mathrm{CPI}(t_{n+1})}{\mathrm{CPI}(t_n)}\right)^2 \tag{23}$$

and this is the only relevant measure. In our case we obtain:

$$4.9914 \qquad (24).$$

We used the simplest function for the approximation of rate of inflation we considered yet, and it gives us far the best approximation we have obtained!

4. Conclusion:

Our sketch brings an open problem: what does the space of solution of (21) look like and how can it help us to understand what kind of measure is this one which we called rate (of interest or inflation. . .). It shows us the possibility approximate the measure of inflation except of index of prices and it suggests, that the solution should be better.

5. Apendix: Maple programs for computation. Suppose that in iota we have function for approximation, in our case:

```
> iota:=.4944210e-1 + .9094682e-2 * cos(x*Pi) + .1215932e-1 * sin(2*x*Pi) + .1823419e-1 *
sin(4*x*Pi)+ .5263253e-1 * sin(1/3*x*Pi) - .9667420e-2 * cos(1/3*x*Pi) - .7212055e-1 * cos(1/4*x*Pi)+
.2834145e-1 * sin(1/2*x*Pi);
```

Suppose, that values of CPI are in the file Values and the corresponding moments of time, in which are values measured, are in file Points. Then we can compute the preciseness of approximation using following program:

```
> Mistake := 0;
> for i to NumberOfPoints do
> A := evalf(int(ln(1+iota),x = Points[1] ..  Points[i+1]));
> A := Values[1]*exp(A);
> Mistake := Mistake+(A-Values[i+1])^2
> end do:
> print(Sum('Delta'^2,i)=Mistake);
```

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